Lesson Plan 1: "Up-Periscope" – Distance to the Horizon

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Instructional Goal

This lesson is designed for Algebra and Geometry teachers looking for an application activity on ratios and proportions and the Pythagorean Theorem , to find the distance between two points.

Common Core State Mathematics Standards:

- HSG-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- HSG-SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.
- HSF-TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context (for Activity 3)

The students will learn about the periscopes used on submarines and how to estimate how far away a vessel or landmark or other physical structure is when viewed in the periscope using ratios and proportions. Below, a submarine with both the number one (attack) and number two (electronic) periscopes raised, with top of the sail exposed and making a visible trailing wake. Modern submarines use 360 degree view mast-mounted cameras, instead of hull-penetrating optical periscopes.

Students will also use the Pythagorean Theorem to calculate the distance they are from a vessel or object on the horizon.

(Extension) The students will use the inverse sine function to determine the arc length (distance) from the submarine to the object on the horizon.

Background Knowledge

Students should understand and be able to apply the Pythagorean Theorem to a right triangle and find a missing side length.

Students should also be able to write proportions and find a missing measurement.

Finally, students should be able to find the length of a segment of a circle by applying the Arc Length ratio.

- (Length of Arc = product of the ratio of the measure of the arc to 360 degrees and the circumference of the circle)
- Without knowing the measurement of the angles in the triangle, students will need to use the inverse sine function.

Navy Background

1. **http://www.ussnautilus.org/virtualTour/attack.shtml** for the Submarine Force Museum Nautilus attack center, and for details of the function of the periscope.

2. 360 degree virtual tour of USS Nautilus Periscope Room: http://www.ussnautilus.org/virtualTour/nautilus/_flash/USS%20Nautilus_nautilus.html

Lesson Plan Activity Summaries
(each has its own student worksheet)

Activity 1 Periscope Observations and Student Worksheet 1

I. View videos on submarine periscopes (http://usnavymuseum.org/Ex2_Navigation.asp select Action Below, Underhull Surveillance of a Surface Ship using a submarine periscope) and on youtube at http://www.youtube.com/watch?v=74vYKL3OYAc .

   On today's submarines, modern computers do the calculations for you, but it is still necessary to understand the mathematics behind the calculations. In the earlier days before modern computers, sailors did the mathematical calculations by memory or by hand. See www.navystemfortheclassroom.com for information on the 21st century Navy.

II. Discuss different settings for the periscope magnification: 1.5x and 6x. Discuss what the markings on the viewing screen mean, and how you can determine the distance you are from something you view in the periscope by using the markings on the screen and knowing what the object is that you are looking at. (Reference is the now declassified Submarine Torpedo Fire Control Manual http://www.hnsa.org/doc/attack/ chapter 5).

III. Determine what you are looking at through the periscope. Determine the height of the viewed object. Gather the information from the periscope screen and set up a proportion to determine how far away the viewed object it.

IV. Discuss how ratios and proportions are used in determining the distance between you and what you see in the periscope screen.

Activity 2 Using the Periscope for Navigation and Student Worksheet 2

I. With the periscope, you can measure the bearing to an object on land and determine the range that object, to determine your position at sea.

II. Using the height of the object being viewed and the height of your periscope above the waterline, draw a picture of your submarine and periscope, the viewed object, and the curve of the Earth.

III. Draw a line connecting the maximum height of both objects. Discuss how that line is a tangent to the Earth.

IV. Draw the radius of the Earth to both objects and to the point where the tangent line touches the Earth.

V. Calculate the missing side lengths of any right triangles, and determine how far you are from the other object.

VI. Discuss how objects with different heights will be seen from different distances away.

Extension/Application Activity 3 Calculating Arc Length and Student Worksheet 3

I. Using the earths radius, your height, and the height of the object viewed, calculate the measurement of the angles formed in the right triangles.

II. Use the ratios of arc length to determine how far you must travel to reach the object viewed.

Activity 1 Periscope Observations

Figure 1

I. **INTRODUCTION**. View the video on submarine periscopes at http://www.youtube.com/watch?v=74vYKL3OYAc and http://usnavymuseum.org/Ex2_Navigation.asp (select Action Below, Under-hull Surveillance of a Surface Ship using a submarine periscope) if not already viewed.

On today's submarines the periscopes are computerized and all the calculations are done for you. However, it is still necessary to understand the mathematics behind the calculations. In the earlier days before modern computers, sailors did the mathematical calculations by memory or by hand.

Read the news story about a submarine that lost its periscope: http://abcnews.go.com/blogs/politics/2013/01/navy-sub-goes-bump-in-the-night-and-loses-its-periscope/

II. **TERMINOLOGY**. (Reference is the now declassified Submarine Torpedo Fire Control Manual http://www.hnsa.org/doc/attack/ chapter 5).

Look at the view from the periscope in Figure 1 above.

- Discuss what the line- markings on the viewing screen (reticle) mean.

The reticule of the periscope has inscribed on it a series of vertical and horizontal lines. In low power each small division represents one degree while in high power each division represents 1/4 degree.

- How can you determine the distance from an object viewed in the periscope by using the markings on the screen and knowing the height of the object that you are looking at?

If the Approach Officer knows or can estimate the masthead height of the target in feet the number of horizontal divisions covered by the ship between its water line and masthead will be a measure of the range of the target.

The obvious solution is some form of pre-computed graph or scale. We know that at a range of 1000 yards, 17 1/2 yards, or 52.5 feet will subtend an angle of 1 degree.

Using this relation we can deduce the following formulas:

R(range) = (19.1 h / n)   For low power

R(range) = (76.2 h / N)   For high power
R= range in yards
h= height in feet of target
n number scale divisions low power
N number scale divisions hi power

- Discuss the 1.5x setting and the 6x settings.
- What are the TB and RB and ELEV labels at the bottom of the screen?
  TB is true bearing relative to the earth's geographic direction
  RB is relative bearing, based on the bow of the ship is 0 degrees, astern is 180 degrees, etc in a circle around the ship.
  ELEV is the elevation or viewing angle relative to the horizontal of zero degrees.

III. Example from http://www.hnsa.org/doc/attack/ for calculating range from scale divisions

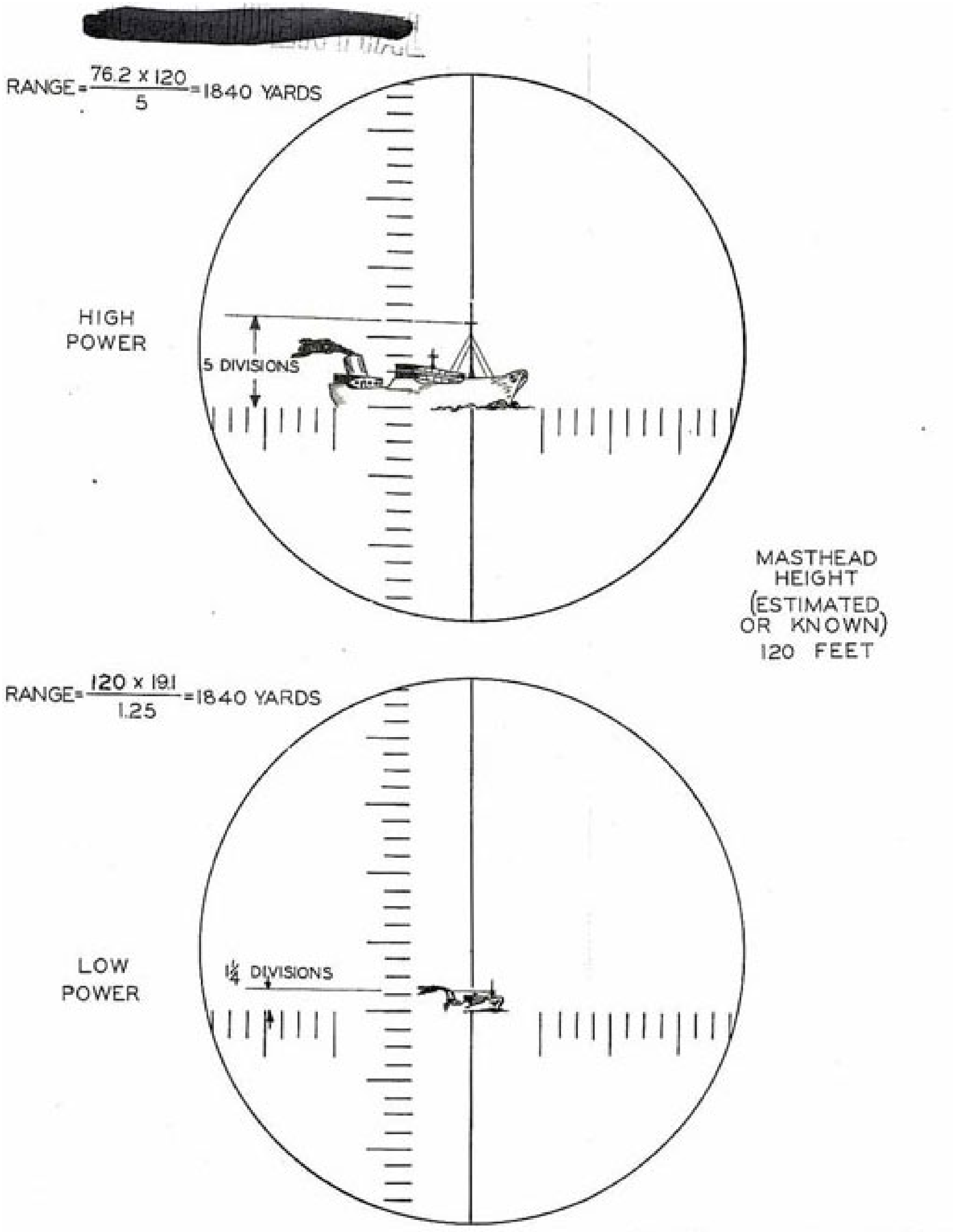
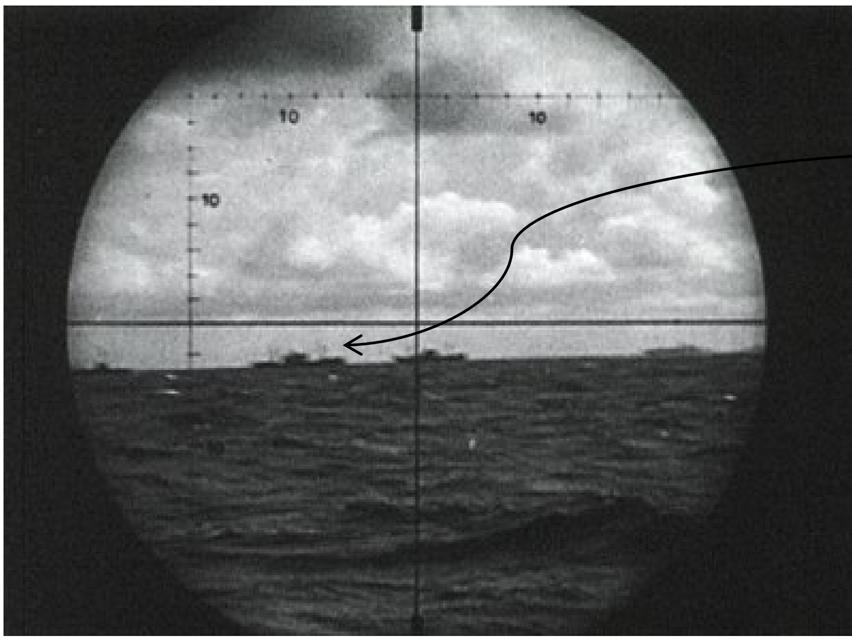


PLATE 11 [redacted]

IV. **Student Worksheet 1**. Using the known dimensions of the vessel you are looking at in figure 2 below and the periscope settings and markings provided, set up the known ratios and any proportions you can to determine how far away the viewed object it.



The vessel in the diagram is a Merchant Ship

It has a mast head height above the water line of 70 feet

The periscope is set in low power, at 1.5x

Figure 2

Range = 19.1 x 70/n

What is n? The number of divisions in the periscope from the waterline of the target to the highest point of its 70 ft mast head height.

Estimate n and calculate your answer.

V. Discuss how ratios and proportions are used in determining the distance between you and what you see in the Periscope screen. How does that distance change with different magnification of the periscope?

Activity 2 Using the Periscope for Navigation

You are in a submarine that is in the Pacific Ocean off of the coast of South America. You look through the periscope and can just see the top of the Christ the Redeemer statute. Use **Student Worksheet 2** to complete the below analysis.

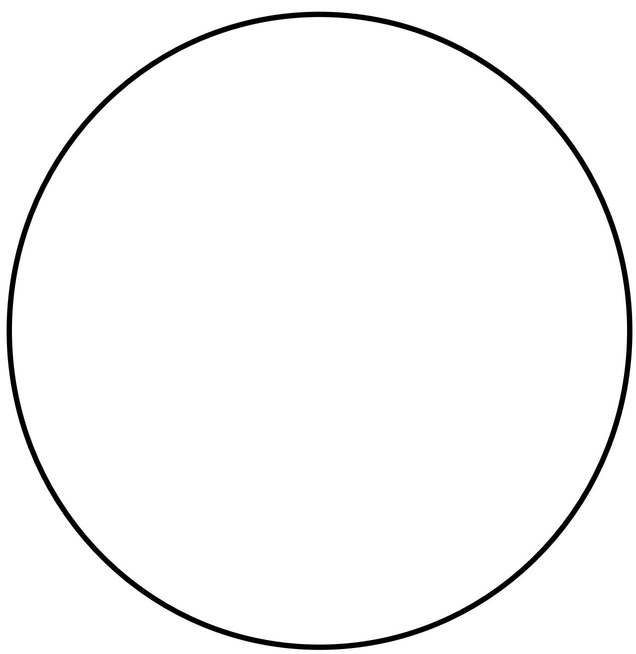
Your computer gives you information that your Commanding Officer (CO) believes to be incorrect. The CO thinks that the computer has malfunctioned and instructs you to manually check the data.

a) Go to http://en.wikipedia.org/wiki/Christ_the_Redeemer_(statue) and find out how high the top of the statue is above the water line.

b) Using that information and the fact that the top of your periscope is 70 ft, draw a circle to represent Earth. Draw a tangent line to the circle.

c) Draw the radius of the Earth to both objects (your periscope top and the top of Christ the Redeemer) and to the point where the tangent line touches the Earth. Label the right angles.

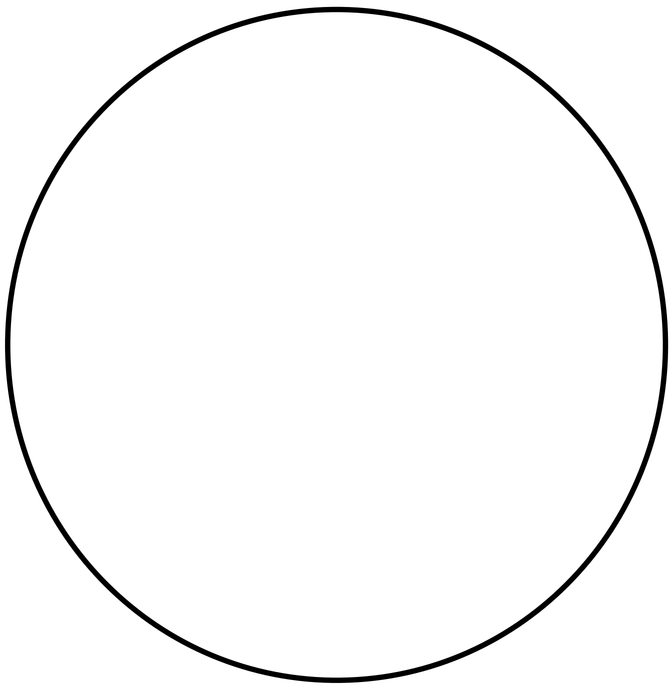
d) Calculate the missing side lengths of the right triangles, and determine how far you are from the other object.



e) Discuss how objects with different heights will be seen from different distances away.

Extension/Application Activity 3, Calculating Arc Length

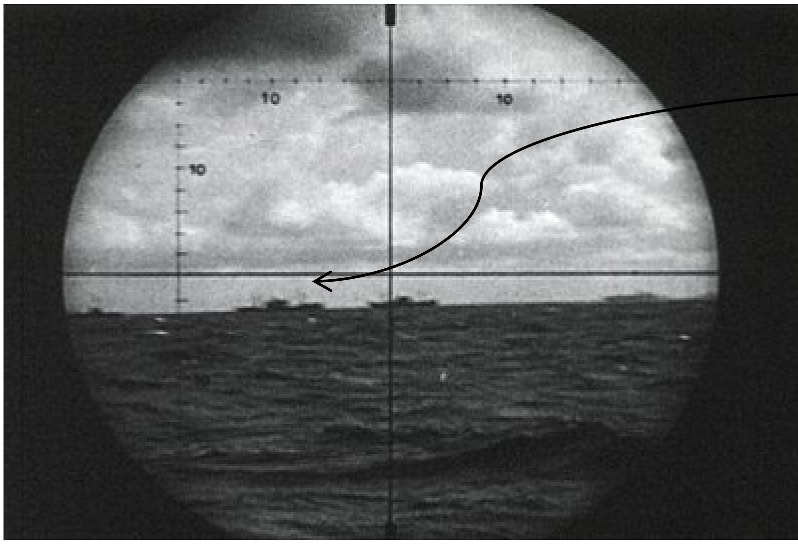
1. Using the Earth's radius, the height of your periscope, and the height of the object viewed, calculate the measurement of the angles formed in the right triangles. Label the circle with all needed information. information   For the object, pick any tall object near a coastline (or the statue in Activity 2).



2. Use the ratios of arc length to determine how far you must travel along the earth's surface (ocean) to reach the object viewed.

Activity 1 Student Worksheet

Figure 2



The vessel in the diagram is an LST

It has a mast head height above the water line of 70 feet

The periscope is set at 1.5x

1. Using the known dimensions of the vessel you are looking at in Figure 2 above and the periscope settings and markings provided, set up the known ratios and any proportions you can to determine how far away the viewed object it.

2. Discuss how ratios and proportions are used in determining the distance between you and what you see in the Periscope screen. How does that distance change with different magnification of the periscope? How does the distance change when the vessel is shorter than 100 feet? taller than 100 feet?

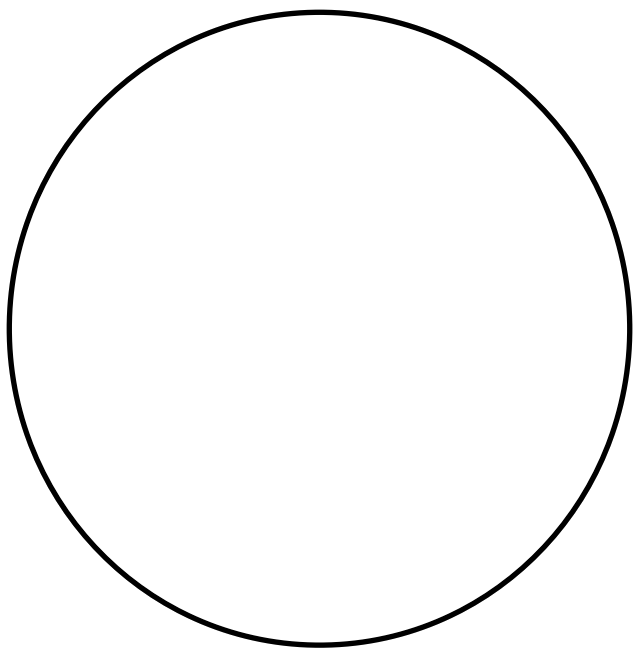
Activity 2 Student Worksheet

1. Go to http://en.wikipedia.org/wiki/Christ_the_Redeemer_(statue) and find out how high the top of the statue is above the water line.

2. Use the circle below to represent Earth. Draw a tangent line to the circle. Draw you submarine and periscope at one endpoint of the tangent line, and the statue of Christ the Redeemer at the other endpoint of the tangent line.

3. Draw the radius from the center of the Earth (the circle) to both objects and to the point where the tangent line touches the Earth. Label the right angles.

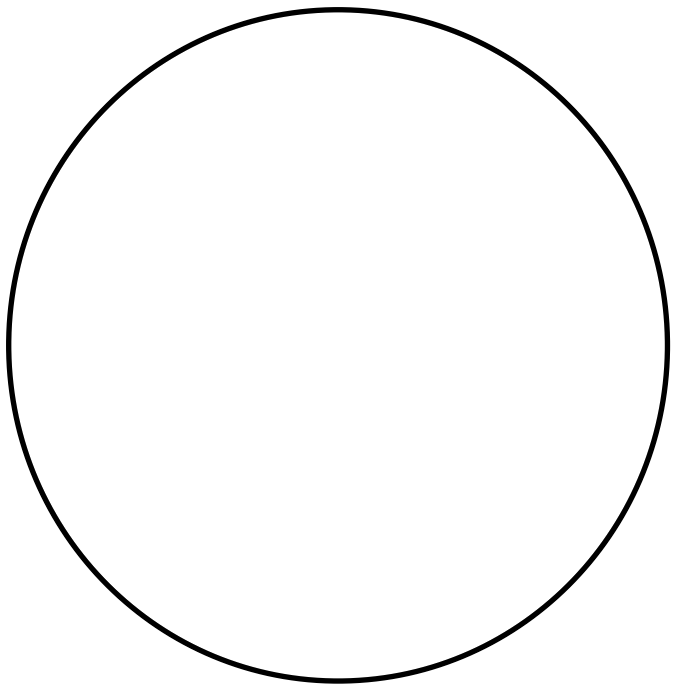
4. Using the Pythagorean Theorem, calculate the missing side lengths of the right triangles, and determine how far you are from the other object.



5. Discuss how objects with different heights will be seen from different distances away.

Activity 3 Student Worksheet

1. Using the Earth's radius, the height of your periscope, and the height of the object viewed, calculate the measurement of the angles formed in the right triangles. Label the circle with all needed information For the object, pick any tall object near a coastline (or the statue in Activity 2).



2. Use the ratios of arc length to determine how far you must travel along the earth's surface (ocean) to reach the object viewed.